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On the General Equations of Electro-magnetic Action, with Application to a New Theory of Magnetic Attractions, and to the Theory of the Magnetic Rotation of the Plane of Polarization of Light.

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In the last number of this Journal I gave what may be considered as the introduction to this paper, and the two should be read continuously.

II. *General Equations in an Extensive Conducting Medium.*

In thinking over the subject there discussed, and seeing how intimately rotation is connected with magnetic phenomena, and how, according to Maxwell's theory of electric displacement, there can be no such thing as an electric current which is not closed, I have been led to a number of curious theorems. And I have been further guided by the idea of Faraday on the conduction of magnetic lines of force, and by the well-known equations of vortex motion which apply to electric and magnetic phenomena. To free my mind from all complications of conductors and non-conductors, and to place magnetic action and electric conduction on the same footing, I conceive of an infinite conducting medium filling all space in the same manner as all space is filled with a conductor of magnetic lines of force, and consider the electro-magnetic action. Such a medium is often used in the theory of electric conduction, but I am not aware that others have occupied themselves much with the magnetic action of currents in such a case. At least I believe the following theory to be new.

It is usual in the ordinary theory of magnetic action to conceive of the poles of the magnet as separated from each other, and to define the magnetic field at any point as the force acting on a unit pole at that point. As analogous to that, let us conceive of points in the medium at which electricity is either generated or destroyed, so that electric currents shall radiate either from or to the points. Let us call these points electric points, and let the same quantity of electricity stream from them whatever their position, and let the strength of the point be denoted by e, where $4\pi e$ is the quantity of electricity streaming *from* it. Let the strength of the magnetic poles be denoted by m, which will be + for North polarity and — for South. This system is perfectly symmetrical with the other, as $4\pi m$ lines of force stream from magnetic poles of strength m.

AXIOM. Two magnetic poles, two electric points, or a magnetic pole and an electric point, cannot exert force on each other except in the direction of the line joining them, for they are symmetrical around that line.

Thus to find the action between an electric point and a magnetic pole, consider any two current rays situated symmetrically with respect to the pole: the true magnetic action of these two will be zero, and hence the force to and from the point will be zero. But the ends of these rays may still have an unknown action. One end of these rays is at the electrical point, and the other on the surface of the sphere at an infinite distance: the action of the sphere is zero and hence there may be a force between an electrical point and a magnetic pole with which we are yet unacquainted, and whose existence we are almost unable to prove experimentally, seeing that we can hardly experiment on unclosed circuits. But there is no true magnetic action such as we are acquainted with. This conducts us to the following remarkable proposition, which is extremely useful.

PROP. The *true** magnetic action of any system of currents which can be generated in an unlimited medium by electric points is zero. Or, in other words, the magnetic action of any system of electric currents which are acyclic is zero.

In an unlimited medium, then, unclosed electric currents have no magnetic action. According to Maxwell's theory, unclosed circuits cannot exist; but on the ordinary theory, the discharge of conductors produces such currents.

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* By *true* magnetic action I mean such magnetic action as we detect in *closed* circuits, and do not include the direct attraction or repulsion between an electric point and a magnetic pole, which we have seen above *may* exist in the case of unclosed circuits. When speaking of magnetic action, I shall generally mean *true* magnetic action.

Let us now conceive of a plus and a minus electric point placed very near each other. Any series of such points can have no magnetic action. But now let us join the points by a line, and suppose such an electro-motive force to exist along the line as to produce a current equal to the strength of one of the points. The current in this line can then have magnetic action; and as the whole circuit is closed, we can calculate it by the well-known formula for the element of a closed circuit. For the unknown direct action between the pole and the electric points will then vanish, because the currents are closed, and there are no ends to the lines of flow. So that the whole magnetic action in the medium will be that due to the current along this line alone. A current can only be produced along such a line by the action of an electro-motive force along it.

Hence we have the remarkable proposition, that in such a medium the action reduces itself to an action between magnets and *electro-motive forces*, instead of between magnets and ordinary *electric currents*. And all the equations for the magnetic action of such currents are much simplified.

To represent the conduction of currents along wires of any shape in such a medium, we have only to suppose the electro-motive force to be in the direction of the wires, and by a proper distribution of electro-motive force, all cases of conduction in limited media can be represented.

Let us again conceive of a plus and a minus magnetic pole near each other in a stream of electricity coming from electric points. They will evidently not have any tendency to a motion of translation in any direction. But now let the poles be joined so that the lines of force flow from one to the other, and the magnet would probably tend to move across the currents. Hence, reasoning as before, this kind of magnetic action cannot be explained, unless the lines of magnetic induction form closed circuits. We cannot speak definitely about this case, as we cannot experiment on anything but closed circuits. But the action seems probable.

From the analogy with the case of electric conduction, let us suppose such a force to exist between the two magnetic points as to cause the same number of lines of induction to pass between the points as flow out of either of them, and call this force a *magneto-motive force*. Such a force is proportional to what has hitherto been called the magnetization, and replaces with a definite mathematical idea the old indefinite idea of *coercive force*.

As thus all lines of magnetic induction and all electric currents are closed, we have to deal in either case with cycles and cyclic regions. Hence the equations of vortex motion should apply to these cases.

Again, when a potential is cyclic, we know very well that we can divide the region into acyclic regions by diaphragms, and that the potential in each of these regions can be represented by integrals taken over the diaphragms enclosing it.

With the ideas thus introduced, all equations giving the relations of *electric currents* to *electro-motive force* are exactly similar to the equations giving the relations of *magnetic induction* to *magneto-motive force.* All Thomson's ideas with respect to lamellar and solenoidal distribution of magnetism thus apply to electro-motive force. If we conceived electro-motive force distributed within a certain region similar to a magnet and proportional to the magnetization, the electric currents at every point of space would be proportional to the magnetic induction in the case of the magnet, without any limitation as to the point being within or without the magnet.

Should the components of the electro-motive force satisfy the lamellar or solenoidal condition within the region, which condition is the same as the equation of continuity, then a single diaphragm enclosing the region will divide space into two acyclic regions, and the electric currents within and without the region can be represented by integrals over this surface. The distribution of electric points for the outside integration must be similar to that of the so-called surface distribution of magnetism on the magnet.

The magnetic action of an electric current is well known to be equivalent to that of a magnetic shell with its edge in the current. Such a magnetic shell merely consists of a sheet of some substance magnetized in a direction normal to the surface. In terms of the words used in this paper, it is a sheet of matter possessing a magneto-motive force in a direction normal to the surface.

As there must be a similar and analogous theorem in the other direction, let us introduce the idea of an electro-motive shell, that is, a shell of matter which possesses an electro-motive force in a direction normal to its surface. The potential of the electric currents from such a shell is evidently equal to the solid angle subtended by the shell, as in the analogous case in magnetism.

Let us now develop the reciprocal relations of electro-motive force and magneto-motive force in a conducting medium. Let us call certain small regions possessing electro-motive or magneto-motive forces, electro-motive or magneto-motive points: these are similar to elements of currents or magnets, ordinarily so called. We then have the following reciprocal theorems.

An electro-motive point tends to pass across the lines of magnetic force in a direction at right angles to both, and therefore a magnetic pole tends to revolve around the electro-motive point.

Reciprocally, a magneto-motive point tends to pass across the electric current in a direction at right angles to both, and therefore an electric point tends to revolve around the magneto-motive point.

Could we prove that an electric point tends to move with the electric current in the same manner as a magnetic pole tends to move in the direction of the lines of magnetic force, the reciprocity would be more exact.

The forces which act or may act between two electric points may be divided into two classes, the electrostatic and electromagnetic forces. The electrostatic forces are evidently the same in the given system as if we should gradually reduce the conductivity to zero, the potential remaining the same. Hence the force between two electric points of strengths m and m' will be

$$\frac{Kmm'}{\mu'^2 r^2},$$

where K is the specific inductive capacity and μ' is the conductivity.

The calculation of the electromagnetic forces depends upon the existence of a force of tension along the current which will be produced when the elements have some action on each other in the direction of their length. As we are unable to prove the existence of such a force, we are unable to say whether two electric points attract each other electromagnetically or not.

That the reciprocity is not perfect, we also prove as follows. Each magneto-motive element is equivalent to an element having an electric current around it, or rather an electro-motive force around it; in other words, the magneto-motive force is proportional to what Maxwell calls the *curl* of the electro-motive force. But the lines of magnetic force around an element of electro-motive force fill all space according to a well-known law, and are not merely packed closely around the element.

The electric potential due to an electro-motive force, A', in an element, $dxdydz$, is, for all points outside the element, as we shall see further on,

$$V = A' \frac{\cos\theta}{r^2}\, dxdydz,$$

and the magnetic force

$$\mathfrak{H} = - A' \frac{\sin\theta}{r^2}\, dxdydz;$$

where θ is the angle between the radius vector and the axis of the electro-motive force. It is evident that these can be both calculated from the same function, for let us define a quantity, U, by the equation,

$$U = A' \frac{1}{r}\, dxdydz;$$

then

$$V = -\frac{dU}{dx} \quad \text{and} \quad \mathfrak{H} = \frac{dU}{dg},$$

where

$$g = \sqrt{y^2 + z^2}.$$

Proceeding in this way we can deduce the whole theory of vector potentials as applied to this case, but with this difference from the ordinary theory that the vector potential is given with reference to the *electro-motive* force rather than to *electric currents*.

But I prefer to treat the subject in a manner similar to vortex motion, and symmetrically. I shall first give the theory for electric conduction and for magnetism separately, and then introduce the electro-magnetic relations of the two. Having thus developed the old theory, I shall then consider the modification required by the newly discovered action.

Let a, b, c be the components of the magnetic induction.

Select three quantities, F, G, and H, to satisfy the equations

$$a = \frac{dH}{dy} - \frac{dG}{dz}$$

$$b = \frac{dF}{dz} - \frac{dH}{dx}$$

$$c = \frac{dG}{dz} - \frac{dF}{dy};$$

whence

$$\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} = 0.$$

These equations are satisfied by taking any three quantities L, M, and N, such that

$$F = \frac{dN}{dy} - \frac{dM}{dz}$$

$$G = \frac{dL}{dz} - \frac{dN}{dx}$$

$$H = \frac{dM}{dx} - \frac{dL}{dy};$$

whence

$$\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} = 0.$$

Hence, putting

$$J = \frac{dL}{dx} + \frac{dM}{dy} + \frac{dN}{dz},$$

Let a', b', c' be the components of the electric current.

Select three quantities, F', G', and H', to satisfy the equations

$$a' = \frac{dH'}{dy} - \frac{dG'}{dz}$$

$$b' = \frac{dF'}{dz} - \frac{dH'}{dx}$$

$$c' = \frac{dG'}{dz} - \frac{dF'}{dy};$$

whence

$$\frac{da'}{dx} + \frac{db'}{dy} + \frac{dc'}{dz} = 0.$$

These equations are satisfied by taking any three quantities, L', M', and N', such that

$$F' = \frac{dN'}{dy} - \frac{dM'}{dz}$$

$$G' = \frac{dL'}{dz} - \frac{dN'}{dx}$$

$$H' = \frac{dM'}{dx} - \frac{dL'}{dy};$$

whence

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dz} = 0.$$

Hence, putting

$$J' = \frac{dL'}{dx} + \frac{dM'}{dy} + \frac{dN'}{dz},$$

we have

$$a = -\Delta^2 L + \frac{dJ}{dx}$$

$$b = -\Delta^2 M + \frac{dJ}{dy}$$

$$c = -\Delta^2 N + \frac{dJ}{dz}.$$

Whence, if we choose some other quantity, χ, so that

$$\chi = \frac{1}{\mu}\iiint \frac{J}{r}\, dxdydz,$$

we may write

$$L = \frac{1}{4\pi}\iiint \frac{a}{r}\, dxdydz - \mu\frac{d\chi}{dx}$$

$$M = \frac{1}{4\pi}\iiint \frac{b}{r}\, dxdydz - \mu\frac{d\chi}{dy}$$

$$N = \frac{1}{4\pi}\iiint \frac{c}{r}\, dxdydz - \mu\frac{d\chi}{dz}.$$

$$\text{Let } p = \frac{1}{r}.$$

Substituting these quantities in the previous equations, we have, as χ disappears,

$$F = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pc)}{dy} - \frac{d\,(pb)}{dz}\right\} dxdydz$$

$$G = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pa)}{dz} - \frac{d\,(pc)}{dx}\right\} dxdydz$$

$$H = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pb)}{dx} - \frac{d\,(pa)}{dy}\right\} dxdydz$$

$$F = \frac{1}{4\pi}\iiint\left\{p\left(\frac{dc}{dy} - \frac{db}{dz}\right) + c\frac{dp}{dy} - b\frac{dp}{dz}\right\} dxdydz$$

$$G = \frac{1}{4\pi}\iiint\left\{p\left(\frac{da}{dz} - \frac{dc}{dx}\right) + a\frac{dp}{dz} - c\frac{dp}{dx}\right\} dxdydz$$

$$H = \frac{1}{4\pi}\iiint\left\{p\left(\frac{db}{dx} - \frac{da}{dy}\right) + b\frac{dp}{dx} - a\frac{dp}{dy}\right\} dxdydz.$$

we have

$$a' = -\Delta^2 L' + \frac{dJ'}{dx}$$

$$b' = -\Delta^2 M' + \frac{dJ'}{dy}$$

$$c' = -\Delta^2 N + \frac{dJ'}{dz}.$$

Whence, if we choose some other quantity, χ', so that

$$\chi' = \frac{1}{\mu'}\iiint \frac{J'}{r}\, dxdydz,$$

we may write

$$L' = \frac{1}{4\pi}\iiint \frac{a'}{r}\, dxdydz - \mu'\frac{d\chi'}{dx}$$

$$M' = \frac{1}{4\pi}\iiint \frac{b'}{r}\, dxdydz - \mu'\frac{d\chi'}{dy}$$

$$N' = \frac{1}{4\pi}\iiint \frac{c'}{r}\, dxdydz - \mu'\frac{d\chi'}{dz}.$$

$$\text{Let } p = \frac{1}{r}.$$

Substituting these quantities in the previous equations, we have, as χ disappears,

$$F' = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pc')}{dy} - \frac{d\,(pb')}{dz}\right\} dxdydz$$

$$G' = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pa')}{dz} - \frac{d\,(pc')}{dx}\right\} dxdydz$$

$$H' = \frac{1}{4\pi}\iiint\left\{\frac{d\,(pb')}{dx} - \frac{d\,(pa')}{dy}\right\} dxdydz$$

$$F' = \frac{1}{4\pi}\iiint\left\{p\left(\frac{dc'}{dy} - \frac{db'}{dz}\right) + c'\frac{dp}{dy} - b'\frac{dp}{dz}\right\} dxdydz$$

$$G' = \frac{1}{4\pi}\iiint\left\{p\left(\frac{da'}{dz} - \frac{dc'}{dx}\right) + a'\frac{dp}{dz} - c'\frac{dp}{dx}\right\} dxdydz$$

$$H' = \frac{1}{4\pi}\iiint\left\{p\left(\frac{db'}{dx} - \frac{da'}{dy}\right) + b'\frac{dp}{dx} - a'\frac{dp}{dy}\right\} dxdydz.$$

These equations, as we have thus obtained them, do not contain any of the phenomena of electro-magnetism, but each series of equations is entirely separate from the other series. The only assumption we have made is that a, b, and c, as well as a', b', and c', are vector quantities which satisfy the equation of continuity at *every* point of space. And this we know to be true in magnetism, where we cannot separate the North from the South pole of a magnet, and in the case of electric currents, provided we also include the displacement currents. If we could separate the poles of a magnet, or if we consider the electric currents from the discharge of electrified bodies without the currents of displacement, there will always be certain points where the lines of force or the currents end, and where, therefore, the equation of continuity does not apply. Both the lines of force, then, and the electric currents, must, as we have shown, be cyclic; and they can have no real potential in general, though by the proper placing of diaphragms we may cause the space to be acyclic, and so may express the potential by a proper integration taken over the diaphragms. The value of the potential so obtained will then apply to all space which can be reached without passing through a diaphragm.

Hence the integrals which we have given can be expressed either as surface integrals over some surface or surfaces, or as volume integrals throughout some volume occupied by the sources of the lines of induction or the electric currents. And this last proposition is evidently true from a physical point of view.

Commencing with the electric currents, let us suppose that an electro-motive force, A', acts throughout the element $dxdydz$, in the direction of x. The electric currents will then stream through the cube in the direction of x, and pass out through one end to return through space to the other end, thus completing a cycle.

By enclosing the element within a surface, we divide cyclic space into two acyclic regions. Within the cube the current will evidently be, provided we take a proper unit for measuring the electro-motive force,

$$4\pi\mu' A'.$$

The amount passing out each end of the cube is therefore

$$4\pi\mu' A' dydz\,;$$

and the current potential at every point situated outside the cube and at a very great distance, r, from it will therefore be